Quantile Impulse Response Functions*

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Abstract

A framework to think about and estimate the impact of structural shocks in a multivariate dynamic quantile model is proposed. The key element of the framework is the construction of reduced form quantile shocks, defined as the ratio between a demeaned random variable and its quantile. Structural shocks are obtained from the reduced form quantile shocks by imposing a Choleski type identification assumption: shocks to one random variable may have a contemporaneous impact on the other random variables, but not vice versa. Structural quantile impulse response functions and their asymptotic confidence intervals are obtained. The model is estimated for a series of bivariate models, documenting the relationship between the daily returns of S&P 500 and selected U.S. financial institutions. The estimates reveal the presence of significant asymmetries in the impact and transmission of negative/positive market shocks to the system.

Keywords: Regression quantiles; CAViaR; VAR for VaR.

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1 Introduction

Vector autoregressive (VAR) models are ubiquitously used in econometrics to capture the linear interdependecies among the first moments of multiple time series. They can be easily estimated using ordinary least squares and it is possible to give them a structural interpretation, by imposing restrictions on the variance-covariance matrix of the error terms. White, Kim and Manganelli (2010, 2015) have proposed VAR models to capture the interdependencies among the quantiles — rather than the mean — of multiple time series. They generalise the conditionally autoregressive value-at-risk (CAViaR) model of Engle and Manganelli (2004) to a multivariate framework and use quantile regression, introduced by Koenker and Bassett (1978), for estimation and inference of the unknown parameters. Since their empirical application is aimed at the estimation of the value-at-risk of several financial institutions, they refer to this class of models as VAR for VaR.

A major challenge when using quantile regressions is represented by the analysis of the dynamic properties of the estimated model. When the model is extended to the VAR framework, no joint distribution (and therefore no underlying correlation structure) is available, because quantile regression is a semi-parametric technique which does not require the specification of the full distribution of the error terms. As also emphasised by White, Kim and Manganelli (2015), it remains unclear how to think about structural shocks in the first place in a multivariate quantile regression setup. This paper provides an answer to these challenges.

We introduce a new definition of quantile structural shocks and derive the quantile impulse-response functions (QIRFs) associated with them. To make things concrete, we proceed by setting up a model of the interaction between two random variables, whose tail dynamics may depend on lagged realisations of both random variables. We recover the *reduced form quantile shocks* as the original random variables standardised by their respective quantiles. This standardisation is regularly used in volatility models like generalized autoregressive conditional heteroskedasticity (GARCH), to test whether the model has removed all second moment dynamics from the data. Given the reduced form quantile shocks, we obtain the *structural quantile shocks* by imposing that one of the shocks may have a contemporaneous impact on the other shock but not vice versa. Specifically, we have in mind a situation where shocks to the returns of a stock market index may have a contemporaeous impact on its individual components. This, for instance, is how Brownlees and Engle (2017) identify the impact of a large decline in the stock market value on the recapitalisation needs of financial institutions. Given the structural quantile shocks, it becomes possible to define the quantile impulse response functions as the difference between the expected quantile dynamics with and without the structural shock.

The first part of the paper introduces the system of the conditional quantiles and shows how the reduced form quantile shocks are related to the shocks of an analogous system modeled with multivariate GARCH. In the second part we introduce a formal definition of the QIRFs and develop the associated asymptotic theory for inference. We also propose a measure for asymmetries in QIRFs between the left and right tails.

The last part of the paper presents the empirical estimates of the model on a selected sample of US banks. We first show with a Monte Carlo study how the reduced form quantile shocks pass standard specification tests of no autocorrelation in levels and squares and behave similarly to the standardized residuals obtained from GARCH models. The Monte Carlo experiment also shows that it is possible to accurately recover measures of correlations from the reduced form quantile shocks. These correlation measures are necessary to estimate the structural shocks of the model and derive the associated quantile impulse response functions. We observe important differences across financial institutions in the magnitude and persistence of QIRFs in response to structural shocks to the market index. The increase in risk of financial institutions tends to be much more severe and long lasting than the corresponding increase in upside opportunities, in response to identical market shocks. Such a nuanced finding would be impossible with standard VAR impulse response analyses.

The paper is organized as following. Section 2 defines the bivariate conditional quantile system and links it to the GARCH framework. Section 3 contains definition and construction of the QIRFs. Section 4 provides the asymptotic properties. The Monte Carlo simulation experiment is carried out in Section 5. Empirical application to the equity market returns are presented in Section 6. Section 7 briefly concludes. Technical proofs and analytical results are provided in the Appendix.

2 The model

Consider a set-up with two sequences of random variables $\{y_{mt}\}_{t=1}^{T}$ and $\{y_{it}\}_{t=1}^{T}$ with continuous distribution function, representing for instance returns of a market index and a financial institution. We first define the concept of quantile shock and then develop a framework for quantile impulse response functions (QIRFs).

2.1 Conditional quantile shocks

We introduce the following data generating process:

Definition 1 *(**Quantile DGP**) Let $\mathcal{F}_{t-1}$ be the information set generated by a sequence of financial returns available at time $t-1$, $\{y_{mk}\}_{k=1}^{t-1}$ and $\{y_{ik}\}_{k=1}^{t-1}$. Consider also a confidence level $\theta \in (0,1)$ and the functions q_{mt} and q_{it}, which depend on variables belonging to $\mathcal{F}_{t-1}$. The quantile DGP is:*

$$\begin{aligned} y_{mt} &= q_{mt}\varepsilon_{mt}, \\ y_{it} &= q_{it}\varepsilon_{it}, \end{aligned} \tag{1}$$

where $q_{jt} \neq 0$ and $\Pr(\text{sgn}(q_{jt})\varepsilon_{jt} \leq \text{sgn}(q_{jt})|\mathcal{F}_{t-1}) = \theta$ for $j \in (m,i)$.

*We refer to ε_{jt} as to the **reduced form quantile shocks**.*

A few comments are in order. First, q_{jt} is the quantile of y_{jt}, as $\Pr(y_{jt} \leq q_{jt}|\mathcal{F}_{t-1}) = \Pr(\text{sgn}(q_{jt})\varepsilon_{jt} \leq \text{sgn}(q_{jt})|\mathcal{F}_{t-1}) = \theta$, since $x = \text{sgn}(x)|x|$. Second, it is necessary to keep track of the sign of the quantile, as — unlike in a location-scale model — when standardizing the dependent variable by its quantile, the inequality defining the quantile process depends on it. Third, the quantile DGP is silent about the comovement between the two variables. Fourth, it is not possible to construct a DGP from (1) when $q_{jt} = 0$, $j \in (m,i)$.

To understand the relationship between our quantile DGP and more conventional location-scale DGPs, consider the following DGP where we set the location equal to zero:

Definition 2 *(**Scale DGP**) Let $\mathcal{F}_{t-1}$ be the information set generated by a sequence of financial returns available at time $t-1$, $\{y_{mk}\}_{k=1}^{t-1}$ and $\{y_{ik}\}_{k=1}^{t-1}$. Consider also σ_{mt} and σ_{it}, which are positive valued functions of variables belonging to $\mathcal{F}_{t-1}$. The scale DGP is:*

$$y_{mt} = \sigma_{mt}w_{mt}, \quad y_{it} = \sigma_{it}z_{it}, \tag{2}$$

$$z_{it} = \rho_i w_{mt} + \sqrt{1-\rho_i^2} w_{it}, \quad \rho_i \in [-1, 1], \tag{3}$$

where w_{mt} and w_{it} are i.i.d.(0,1) with continuous cumulative distribution function $F(\cdot)$.

*We refer to w_{mt} and w_{it} as to the **structural shocks**.*

The following proposition establishes the relationship between reduced form quantile shocks and structural shocks.

Proposition 1 ***(Relation between reduced form quantile shocks and structural shocks)*** *Given definitions 1 and 2, the reduced form quantile shocks ε_{mt} and ε_{it} are related to the structural shocks w_{mt} and w_{it} as follows:*

$$\varepsilon_{mt} = \frac{w_{mt}}{F^{-1}(\theta)}, \quad \textit{and} \quad \varepsilon_{it} = \rho_i \varepsilon_{mt} + \sqrt{1-\rho_i^2} \frac{w_{it}}{F^{-1}(\theta)}. \tag{4}$$

Proof 1 *See Appendix A.1.*

We can now introduce the definition of a structural quantile shock.

Definition 3 ***(Structural quantile shocks)*** *Let w_{mt} and w_{it} be as in definition 2, with cdf $F(\cdot)$. The structural θ-quantile shocks for $\theta \in (0,1)$ are defined as:*

$$\eta_{mt} = \frac{w_{mt}}{F^{-1}(\theta)}, \quad \textit{and} \quad \eta_{it} = \frac{w_{it}}{F^{-1}(\theta)}. \tag{5}$$

Our goal in this paper is to recover the structural quantile shocks (η_{mt}, η_{it}) from the reduced form quantile shocks $(\varepsilon_{mt}, \varepsilon_{it})$. Note that given the identification assumption implicit in definition 2, the market reduced form quantile shock and the quantile structural shock coincide, that is $\varepsilon_{mt} = \eta_{mt}$.

The following proposition is an immediate consequence of Proposition 1.

Proposition 2 *Let ε_{mt} be the market quantile structural shock and ε_{it} be the institution quantile reduced form shock. The structural quantile shock associated with the individual financial institution i can be recovered in a second step procedure, by exploiting the correlation structure (4):*

$$\varepsilon_{it} = \rho_i \varepsilon_{mt} + \varsigma_i \eta_{it}. \tag{6}$$

In the light of definition 2, one can interpret this relation in a structural way, so that the individual structural shock η_{it} and the structural market shock $\eta_{mt} = \varepsilon_{mt}$ have a contemporaneous impact on individual reduced-form quantile shock ε_{it}, but individual structural shocks have no contemporaneous impact on market returns.

This model is consistent with the constant conditional correlation model by Bollerslev (1990). Similarly to Engle, Ito, and Lin (1990) and Lin (1997), the tail shock propagates through the dynamic volatility structure and not the time-varying dynamic correlation structure. Although there are many ways to specify a time-varying correlation, here we focus for simplicity on the constant correlation model.

2.2 Conditional quantile functions

In order to study empirically the time-varying tail behavior, we need to introduce a specific data generating process for the time-varying quantiles q_{mt} and q_{it}. We assume the following specification for the quantile functions:

$$\begin{aligned} q_{mt} &= \omega_m + \beta_m q_{mt-1} + \alpha_m |y_{mt-1}|, \\ q_{it} &= \omega_i + \beta_i q_{it-1} + \alpha_i |y_{it-1}| + \alpha_{im} |y_{mt-1}|, \end{aligned} \tag{7}$$

where the market quantile function q_{mt} follows the univariate CAViaR model by Engle and Manganelli (2004) and the individual quantile function q_{it} is extended to include spillovers from the lagged market rate y_{mt-1}. If $\alpha_{im} = 0$, the quantile q_{it} reduces to the symmetric absolute value CAViaR specification. This formulation allows the two specifications to be estimated independently from each other and therefore provides substantial computational simplicity.

This particular specification would be consistent with the following daily volatility components by Taylor's (1986) linear GARCH (1,1)

$$\begin{aligned} \sigma_{mt} &= \tilde{\omega}_m + \beta_m \sigma_{mt-1} + \tilde{\alpha}_m |y_{mt-1}|, \\ \sigma_{it} &= \tilde{\omega}_i + \beta_i \sigma_{it-1} + \tilde{\alpha}_i |y_{it-1}| + \tilde{\alpha}_{im} |y_{mt-1}|, \end{aligned} \tag{8}$$

where the parameters of the conditional quantile processes in (7) are given by $\alpha_j = \tilde{\alpha}_j F^{-1}(\theta)$, $\omega_j = \tilde{\omega}_j F^{-1}(\theta)$ for $j \in (m, i)$ and $\alpha_{im} = \tilde{\alpha}_{im} F^{-1}(\theta)$.

3 Quantile impulse response analysis

In this section, we present a framework for analyzing the propagation of a structural quantile shock to the market rate y_{mt} through the system of conditional quantiles (7). For this purpose, we first provide the definition of the quantile impulse response function (QIRF) and then derive its analytical expression.

Definition 4 (QIRF) *The quantile impulse response function for $y_{jt}, j \in (m,i)$ in response to the market structural quantile shock $\varepsilon_{mt} = 1$ at horizon h is defined as*

$$\boldsymbol{\Delta}^h \equiv \mathrm{E}_t[\boldsymbol{q}_{t+h}|\varepsilon_{mt} = 1] - \mathrm{E}_t[\boldsymbol{q}_{t+h}], \quad h = 1, 2, \ldots,$$

where $\boldsymbol{q}_{t+h} = (q_{mt+h}, q_{it+h})'$.

Notice that our definition of quantile shock to the market rate as $\varepsilon_{mt} = 1$ implies a shock to y_{mt} that equals its conditional θ-quantile value q_{mt}. The above definition of the QIRFs, therefore, traces the impact over time of a quantile shock to the market on the quantiles of the market and the individual financial institution. It resembles standard VAR impulse response functions, but instead of looking at how shocks affect the mean it looks at their impact on quantiles. Furthermore, instead of choosing arbitrary shocks, quantile impulse response functions are based on a θ-quantile shock, which is coherent with the estimated θ-quantile model.[1]

Next, we rewrite the system of quantiles in (7) as:

$$\begin{aligned} q_{mt+1} &= \omega_m + \tilde{\varepsilon}_{mt} q_{mt}, \\ q_{it+1} &= \omega_i + \tilde{\varepsilon}_{it} q_{it} + \check{\varepsilon}_{mt} q_{mt}, \end{aligned}$$

where the variables $\tilde{\varepsilon}_{mt} = [\beta_m + \alpha_m \operatorname{sgn}(q_{mt})|\varepsilon_{mt}|]$, $\tilde{\varepsilon}_{it} = [\beta_i + \alpha_i \operatorname{sgn}(q_{it})|\varepsilon_{it}|]$ and $\check{\varepsilon}_{mt} = \alpha_{im} \operatorname{sgn}(q_{mt})|\varepsilon_{mt}|$ are functions of the market and individual tail shocks.[2] Notice that in a

[1]For comparison, following the standard exposition (see, for instance, Section 11.4 in Hamilton (1994)), consider any process written as a vector autoregression, $\boldsymbol{Y}_t = \Phi \boldsymbol{Y}_{t-1} + \boldsymbol{V}_t$, where eigenvalues of $\boldsymbol{\Phi}$ lie inside the unit circle and $\boldsymbol{V}_t$ is the vector of serially uncorrelated random variables. Then as $h \to \infty$, $\boldsymbol{Y}_{t+h} = \boldsymbol{\Phi}^{h+1}\boldsymbol{Y}_{t-1} + \boldsymbol{V}_{t+n} + \cdots + \boldsymbol{\Phi}^h \boldsymbol{V}_t$ will have the infinite order moving average representation. By setting $\boldsymbol{V}_{t+1}, \ldots, \boldsymbol{V}_{t+h}, \boldsymbol{Y}_{t-1}$ and all elements of $\boldsymbol{V}_t$ besides jth, v_{jt}, to zero, the element $(\partial y_{it+h}/\partial v_{jt})\delta$ will give the response of ith observation y_i to the impulse $v_{jt} = \delta$ as a function of h. Furthermore, this approach can be formalized as $\boldsymbol{I}_Y(h, \delta, \mathcal{F}_{t-1}) \equiv \mathrm{E}_{t-1}(\boldsymbol{Y}_{t+h}|v_{jt} = \delta) - \mathrm{E}_{t-1}(\boldsymbol{Y}_{t+h})$.

[2]Given our quantile specification (7), the QIRFs associated with the structural shocks $\varepsilon_{mt} = 1$ are observationally equivalent to those associated with the shock $\varepsilon_{mt} = -1$. More generally, it is not always

model like (2)-(3), the sign of the θ-quantile is always the same for given θ, and in practice it will be always negative for lower tail quantiles like 1% or 5% and always positive for upper tail quantiles like 95% or 99%. The system can be conveniently written in a matrix form as:

$$\begin{pmatrix} q_{mt+1} \\ q_{it+1} \end{pmatrix} = \begin{pmatrix} \omega_m \\ \omega_i \end{pmatrix} + \begin{pmatrix} \tilde{\varepsilon}_{mt} & 0 \\ \check{\varepsilon}_{mt} & \tilde{\varepsilon}_{it} \end{pmatrix} \begin{pmatrix} q_{mt} \\ q_{it} \end{pmatrix},$$

or more compactly as

$$\boldsymbol{q}_{t+1} = \boldsymbol{\omega} + \boldsymbol{\Gamma}_t \boldsymbol{q}_t. \tag{9}$$

To guarantee that QIRFs are well defined, we impose the following stationarity assumption.

Assumption 1 ***(Stationarity)*** *The following matrix has eigenvalues less than one in absolute value:*

$$\boldsymbol{\Gamma} \equiv \mathrm{E}_t\, \boldsymbol{\Gamma}_t = \begin{pmatrix} \beta_m + \alpha_m \operatorname{sgn}(q_{mt})\mu_m & 0 \\ \alpha_{im} \operatorname{sgn}(q_{mt})\mu_m & \beta_i + \alpha_i \operatorname{sgn}(q_{it})\mu_i \end{pmatrix},$$

where $\mu_j = \mathrm{E}_t(|\varepsilon_{jt}|)$, $j \in (m, i)$.

The following theorem derives the QIRFs.

Theorem 1 *Given model* (7) *and the DGP of definition 2, if Assumption 1 holds, the QIRFs of Definition (4) can be computed as:*

$$\boldsymbol{\Delta}^h \quad = \quad \boldsymbol{\Gamma}^{h-1} \left(\mathrm{E}_t\left[\boldsymbol{\Gamma}_t | \varepsilon_{mt} = 1\right] - \mathrm{E}_t\left[\boldsymbol{\Gamma}_t\right]\right) \boldsymbol{q}_t, \quad h = 1, 2, \ldots,$$

where,

$$\mathrm{E}_t\left[\boldsymbol{\Gamma}_t | \varepsilon_{mt} = 1\right] - \mathrm{E}_t\left[\boldsymbol{\Gamma}_t\right] = \begin{pmatrix} \alpha_m \operatorname{sgn}(q_{mt})(1 - \mu_m) & 0 \\ \alpha_{im} \operatorname{sgn}(q_{mt})(1 - \mu_m) & \alpha_i \operatorname{sgn}(q_{it})(\tilde{\mu}_i - \mu_i) \end{pmatrix},$$

$\mu_j \equiv \mathrm{E}_t[|\varepsilon_{jt}|]$, $j \in (m, i)$ *and* $\tilde{\mu}_i = \mathrm{E}_t\, |\rho_i + \varsigma_i \eta_{it}|$.

Proof 2 *See Appendix A.2.*

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straightforward to define impulse response functions for non-linear models, as discussed, for instance, by Gallant et al. (1993), Potter (2000) and Lütkepohl (2008).

Note that, in general, the QIRFs are not available in closed form unless the conditional quantile function q_{jt} has the same sign for all t, as it would be the case for the model (2)-(3). However, in the more general case the QIRFs can still be obtained via simulation.[3]

The effect of the market quantile shocks is not necessarily symmetric, since the impact of the left quantile shock can be different from the right quantile shock. Formally, let $\boldsymbol{\Delta}_{\theta}^{h}$ denote the QIRF associated with a left quantile shock to the market rate ε_{mt} with confidence level $\theta \in (0, 0.5)$ and let $\boldsymbol{\Delta}_{1-\theta}^{h}$ be the QIRF for a right quantile shock with confidence level $1-\theta$. We construct a measure of asymmetry in responses using the following metric

$$\boldsymbol{\delta}^{h} \equiv \boldsymbol{\Delta}_{1-\theta}^{h} + \boldsymbol{\Delta}_{\theta}^{h}, \quad h = 1, 2, \ldots. \tag{10}$$

Symmetric impulse response functions imply that $\boldsymbol{\delta}^{h} = 0$ for $h = 1, 2, \ldots$.

4 Asymptotic distribution of quantile impulse response functions

To estimate the parameters and derive their asymptotic properties, we follow a large literature on the estimation of structural dynamic models by first considering its reduced-form representation given by Definition (1) and the system of conditional quantile functions (7) and then recovering the structural relationship (6) (see, for example, Watson (1994) and Lütkepohl (2005)). This strategy can be interpreted as a two-step approach where we first estimate the vector of quantile parameters $\boldsymbol{\gamma} = (\boldsymbol{\gamma}_m', \boldsymbol{\gamma}_i')'$, with $\boldsymbol{\gamma}_m = (\omega_m, \beta_m, \alpha_m)'$ and $\boldsymbol{\gamma}_i = (\omega_i, \beta_i, \alpha_i, \alpha_{im})'$, and then the scalars ρ_i, μ_m, μ_i and $\tilde{\mu}_i$.

In this section, we derive the asymptotic distribution of the QIRFs $\boldsymbol{\Delta}^{h}$ by exploiting the limiting properties of parameters $(\boldsymbol{\gamma}', \rho_i, \mu_m, \mu_i, \tilde{\mu}_i)'$ and then applying the delta method.[4] Given the conditional quantile restriction in the Definition 1, the parameters of the model $\boldsymbol{\gamma}_j$

[3]Note that for $j \in (m, i)$, we can simulate standardized conditional quantile shocks $(\varepsilon_{jt+1}^{(s)}, \ldots, \varepsilon_{jt+h}^{(s)})|\mathcal{F}_t$, $s = 1, \ldots, S$ by resampling their estimated values $(\hat{\varepsilon}_{j1}, \ldots, \hat{\varepsilon}_{jt})$ and use to construct conditional quantile values $(\boldsymbol{q}_{t+1}^{(s)}, \ldots, \boldsymbol{q}_{t+h}^{(s)})$ given the financial return process in (1). Then QIRFs can be constructed using the Monte Carlo averages and Definition 4.

[4] This strategy has been widely used in the literature in order to establish the asymptotic normality of impulse response functions (see, for example, Lütkepohl (1990)).

can be consistently estimated by solving the following asymmetric loss problem by Koenker and Bassett (1978)

$$\hat{\boldsymbol{\gamma}}_j = \arg\min_{\boldsymbol{\gamma}_j} T^{-1} \sum_{t=1}^{T} [\theta - I\left(u_{jt}(\boldsymbol{\gamma}_j) < 0\right)] u_{jt}(\boldsymbol{\gamma}_j), \quad j \in (m, i), \tag{11}$$

where $u_{jt}(\boldsymbol{\gamma}_j) = y_{jt} - q_{jt}(\boldsymbol{\gamma}_j)$, and $q_{jt}(\boldsymbol{\gamma}_j)$ is specified as in (7). Using the estimated parameter vector $\hat{\boldsymbol{\gamma}}_j$ the reduced form quantile shocks can be recovered as $\hat{\varepsilon}_{jt} = y_{jt}/\hat{q}_{jt}, j \in (i, m)$, and used to estimate the parameter $\hat{\rho}_i = \arg\min_{\rho_i} T^{-1} \sum_{t=1}^{T} \left(\hat{\varepsilon}_{it} - \rho_i \hat{\varepsilon}_{mt}\right)^2$ and the moment values $\hat{\mu}_j = T^{-1} \sum_{t=1}^{T} |\hat{\varepsilon}_{jt}|, j \in (i, m)$ and $\hat{\tilde{\mu}}_i = T^{-1} \sum_{t=1}^{T} |\hat{\rho}_i(1 - \hat{\varepsilon}_{mt}) + \hat{\varepsilon}_{it}|$ defined in Section 3.

In order to establish consistency and asymptotic normality for the estimator $\hat{\boldsymbol{\gamma}} = (\hat{\boldsymbol{\gamma}}'_m, \hat{\boldsymbol{\gamma}}'_i)$ we first note that the system of conditional quantile functions (7) can be viewed as a special case of the framework considered in White, Kim and Manganelli (2015, **WKM** henceforth):

Corollary 1 (Corollary to WKM) *Suppose that assumptions 1-6 of* **WKM** *hold, then the estimator* $\hat{\boldsymbol{\gamma}} = (\hat{\boldsymbol{\gamma}}'_m, \hat{\boldsymbol{\gamma}}'_i)$ *is consistent and asymptotically normal*

$$\sqrt{T} \begin{pmatrix} \hat{\boldsymbol{\gamma}}_m - \boldsymbol{\gamma}_m \\ \hat{\boldsymbol{\gamma}}_i - \boldsymbol{\gamma}_i \end{pmatrix} \sim \mathrm{N}\left(\mathbf{0}, \begin{bmatrix} \boldsymbol{H}_m^{-1} \boldsymbol{J}_m \boldsymbol{H}_m^{-1} & \mathbf{0} \\ \mathbf{0}' & \boldsymbol{H}_i^{-1} \boldsymbol{J}_i \boldsymbol{H}_i^{-1} \end{bmatrix}\right),$$

where for a continuous density function f_{jt} the positive definite matrices $\boldsymbol{H}_j, \boldsymbol{J}_j$, $j \in (m, i)$ are given as

$$\begin{aligned} \boldsymbol{J}_j &= \theta(1-\theta) \lim_{T\to\infty} T^{-1} \textstyle\sum_{t=1}^{T} \nabla_{\boldsymbol{\gamma}_j} q_{jt} \nabla_{\boldsymbol{\gamma}'_j} q_{jt}, \\ \boldsymbol{H}_j &= \lim_{T\to\infty} T^{-1} \textstyle\sum_{t=1}^{T} f_{jt}(q_{jt}) \nabla_{\boldsymbol{\gamma}_j} q_{jt} \nabla_{\boldsymbol{\gamma}'_j} q_{jt}. \end{aligned}$$

Since our model is nested in the one considered by **WKM**, Corollary 1 directly follows from Theorem 2 by **WKM** and therefore will not be discussed. Using Theorem 3 by **WKM** the consistent covariance matrix estimator can be constructed as following

$$\begin{aligned} \hat{\boldsymbol{J}}_j &= \tfrac{1}{T} \textstyle\sum_{t=1}^{T} \left(\theta - I(y_{jt} \le \hat{q}_{jt})\right)^2 \nabla_{\boldsymbol{\gamma}_j} \hat{q}_{jt} \nabla_{\boldsymbol{\gamma}'_j} \hat{q}_{jt}, \\ \hat{\boldsymbol{H}}_j &= \tfrac{1}{2T\hat{h}_T} \textstyle\sum_{t=1}^{T} I(|y_{jt} - \hat{q}_{jt}| \le \hat{h}_T) \nabla_{\boldsymbol{\gamma}_j} \hat{q}_{jt} \nabla_{\boldsymbol{\gamma}'_j} \hat{q}_{jt}, \end{aligned}$$

where $\hat{h}_T$ is a suitably chosen bandwidth.[5]

[5]The construction of $\hat{h}_T$ is discussed in detail in Section 6.1.

Under the regularity conditions given by Newey and McFadden (1994, Theorem 3.4.), the asymptotic distribution of $\hat{\rho}_i, \hat{\tilde{\mu}}_i, \hat{\mu}_j, j \in (m,i)$ is derived in the following propositions.[6] Since the techniques involved are standard we briefly discuss the results in Appendix B.

Proposition 3 (Asymptotic normality of two-step estimates) *Let,* $\hat{\varepsilon}_{jt} = y_{jt}/\hat{q}_{jt}$, $\hat{\mu}_j = T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_{jt}$, $j \in (m,i)$ *and* $\hat{\rho}_i = \arg\min_{\rho_i} T^{-1}\sum_{t=1}^{T}(\hat{\varepsilon}_{it} - \rho_i\hat{\varepsilon}_{mt})^2$. *Then* $\hat{\mu}_j$ *and* $\hat{\rho}_i$ *are consistent and asymptotically normal:*

$$\sqrt{T}\,(\hat{\rho}_i - \rho_i) \sim \mathrm{N}(0, \sigma^2_\rho), \quad \textit{and} \quad \sqrt{T}\,(\hat{\mu}_j - \mu_j) \sim \mathrm{N}(0, \sigma^2_{\mu_j}),$$

with $\sigma^2_\rho = \boldsymbol{H}^{-1}_\rho \boldsymbol{J}_\rho \boldsymbol{H}^{-1\prime}_\rho$, $\boldsymbol{J}_\rho = \mathrm{E}\,(\boldsymbol{\psi}_t\boldsymbol{\psi}_t')$, $\boldsymbol{\psi}_t = (\varepsilon_{mt}[\varepsilon_{it} - \rho_i\varepsilon_{mt}], \nabla_{\boldsymbol{\gamma}_m'} q_{mt}[\theta - I(y_{mt} < q_{mt})], \nabla_{\boldsymbol{\gamma}_i'} q_{it}[\theta - I(y_{it} < q_{it})])'$, $\boldsymbol{H}^{-1}_\rho = \left(H^{-1}_\rho, H^{-1}_\rho \boldsymbol{H}_{\rho,\gamma}\boldsymbol{H}^{-1}\right)$, $H_\rho = \mathrm{E}\,\varepsilon^2_{mt}$, $\boldsymbol{H}^{-1} = \mathrm{diag}\left(\boldsymbol{H}^{-1}_m, \boldsymbol{H}^{-1}_i\right)$, $\boldsymbol{H}_{\rho,\boldsymbol{\gamma}} = \mathrm{E}\left((2\rho_i\varepsilon_{mt} - \varepsilon_{it})\frac{\varepsilon_{mt}}{q_{mt}}\nabla_{\boldsymbol{\gamma}_m'} q_{mt}, -\frac{\varepsilon_{it}\varepsilon_{mt}}{q_{it}}\nabla_{\boldsymbol{\gamma}_i'} q_{it}\right)$, $\sigma^2_{\mu_j} = \boldsymbol{H}_{\mu_j}\boldsymbol{\Omega}_{\boldsymbol{\gamma}}\boldsymbol{H}'_{\mu_j}$, $\boldsymbol{H}_{\mu_j} = \mathrm{E}\left(|\varepsilon_{jt}| q^{-1}_{jt}\nabla_{\boldsymbol{\gamma}'} q_{jt}\right)$, $\boldsymbol{\Omega}_{\boldsymbol{\gamma}} = \mathrm{diag}\left(\boldsymbol{H}^{-1}_m\boldsymbol{J}_m\boldsymbol{H}^{-1}_m, \boldsymbol{H}^{-1}_i\boldsymbol{J}_i\boldsymbol{H}^{-1}_i\right)$ *and* $\boldsymbol{H}^{-1}_j$, $j \in (m,i)$ *as defined in Corollary 1.*

Estimates $\hat{\tilde{\mu}}_i$ depend both on the first step $\hat{\boldsymbol{\gamma}}$ as well as the second step estimates $\hat{\rho}_i$.

Proposition 4 (Asymptotic normality of three-step estimates) *Let* $\boldsymbol{\kappa} = (\boldsymbol{\gamma}_m', \boldsymbol{\gamma}_i', \rho_i)'$, $\hat{\tilde{\mu}}_i = T^{-1}\sum_{t=1}^{T}|\hat{\rho}_i(1 - \hat{\varepsilon}_{mt}) + \hat{\varepsilon}_{it}|$ *and* $\boldsymbol{\Omega}_{\boldsymbol{\kappa}} = \mathrm{diag}\left(\boldsymbol{H}^{-1}_m\boldsymbol{J}_m\boldsymbol{H}^{-1}_m, \boldsymbol{H}^{-1}_i\boldsymbol{J}_i\boldsymbol{H}^{-1}_i, \boldsymbol{H}^{-1}_\rho\boldsymbol{J}_\rho\boldsymbol{H}^{-1\prime}_\rho\right)$. *Then* $\hat{\tilde{\mu}}_i$ *is consistent and asymptotically normal:*

$$\sqrt{T}(\hat{\tilde{\mu}}_i - \tilde{\mu}_i) \sim \mathrm{N}(0, \sigma^2_{\tilde{\mu}}),$$

with $\sigma^2_{\tilde{\mu}} = \boldsymbol{H}_{\tilde{\mu}}\boldsymbol{\Omega}_{\boldsymbol{\kappa}}\boldsymbol{H}'_{\tilde{\mu}}$ *and* $\boldsymbol{H}_{\tilde{\mu}} = \mathrm{E}\left([\rho_i\varepsilon_{mt}q^{-1}_{mt}\nabla_{\boldsymbol{\gamma}_m'} q_{mt}, -\varepsilon_{it}q^{-1}_{it}\nabla_{\boldsymbol{\gamma}_i'} q_{it}, (1 - \varepsilon_{mt})]\,\mathrm{sgn}(\rho_i(1 - \varepsilon_{mt}) + \varepsilon_{it})\right)$.

The sample averages can be used for estimating covariance matrices by plugging estimated factors.[7] We can now derive the asymptotic distribution of the QIRFs.

Proposition 5 (Asymptotic distribution of QIRFs) *Let* $\boldsymbol{\nu} = (\boldsymbol{\gamma}_m', \boldsymbol{\gamma}_i', \mu_m, \mu_i, \tilde{\mu}_i)'$ *denote the parameter vector satisfying* $\sqrt{T}\,(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}) \sim \mathrm{N}(\boldsymbol{0}, \boldsymbol{\Omega}_{\boldsymbol{\nu}})$ *with the covariance matrix* $\boldsymbol{\Omega}_{\boldsymbol{\nu}} = \mathrm{diag}\left(\boldsymbol{\Omega}_{\boldsymbol{\gamma}}, \boldsymbol{H}_{\mu_m}\boldsymbol{\Omega}_{\boldsymbol{\gamma}}\boldsymbol{H}'_{\mu_m}, \boldsymbol{H}_{\mu_i}\boldsymbol{\Omega}_{\boldsymbol{\gamma}}\boldsymbol{H}'_{\mu_i}, \boldsymbol{H}_{\tilde{\mu}}\boldsymbol{\Omega}_{\boldsymbol{\kappa}}\boldsymbol{H}'_{\tilde{\mu}}\right)$ *defined in Corollary 1 and Propositions 3-4. Then using the result from Serfling (1980, p. 122):*

$$\sqrt{T}(\hat{\boldsymbol{\Delta}}^h - \boldsymbol{\Delta}^h) \sim \mathrm{N}\left(\boldsymbol{0}, \boldsymbol{G}\boldsymbol{\Omega}_{\boldsymbol{\nu}}\boldsymbol{G}'\right),$$

[6]Note that, in what follows, $\mathrm{diag}(\boldsymbol{A}_1, \boldsymbol{A}_2, \dots)$ denotes a block diagonal matrix with diagonal blocks being some matrices $\boldsymbol{A}_1, \boldsymbol{A}_2, \dots$.

[7]See the exposition in Section 6.3 of Newey and McFadden (1994).

where $\boldsymbol{G} = \nabla_{\boldsymbol{\nu}'}\boldsymbol{\Delta}^h$ *is a nonzero matrix at* $\boldsymbol{\nu}$.[8]

Inference for the asymmetry in responses defined in (10) can be obtained analogously. First define the parameter vector $\tilde{\boldsymbol{\nu}} = (\boldsymbol{\nu}'_{1-\theta}, \boldsymbol{\nu}'_{\theta})'$ where the components are the vectors of parameters $\boldsymbol{\nu}$ evaluated at quantile values θ and $1-\theta$ for $\theta \in (0, 0.5)$, such that $\sqrt{T}(\hat{\boldsymbol{\nu}}_\theta - \boldsymbol{\nu}_\theta) \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Omega}_{\boldsymbol{\nu}_\theta})$ and $\sqrt{T}(\hat{\boldsymbol{\nu}}_{1-\theta} - \boldsymbol{\nu}_{1-\theta}) \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Omega}_{\boldsymbol{\nu}_{1-\theta}})$. Then, again using the result from Serfling (1980, p. 122) for a non-zero matrix $\boldsymbol{G}_{\tilde{\boldsymbol{\nu}}} = \nabla_{\tilde{\boldsymbol{\nu}}'}\boldsymbol{\delta}^h$ it follows that

$$\sqrt{T}(\hat{\boldsymbol{\delta}}^h - \boldsymbol{\delta}^h) \sim \mathrm{N}\left(\mathbf{0}, \boldsymbol{G}_{\tilde{\boldsymbol{\nu}}}\boldsymbol{\Omega}_{\tilde{\boldsymbol{\nu}}}\boldsymbol{G}'_{\tilde{\boldsymbol{\nu}}}\right),$$

where $\boldsymbol{\Omega}_{\tilde{\boldsymbol{\nu}}} = \mathrm{diag}(\boldsymbol{\Omega}_{\boldsymbol{\nu}_{1-\theta}}, \boldsymbol{\Omega}_{\boldsymbol{\nu}_\theta})$.

5 Monte Carlo Evidence

Our empirical model implies that the market return y_{mt} affects the individual quantile function q_{it} not only through its own lag value y_{mt-1} but also through y_{it-1} via the linear relationship (6). This requires an estimate of the correlation coefficient ρ_i, which is recovered with the two-step procedure discussed in the previous section. In order to evaluate the accuracy of our estimation procedure, this section assesses the finite sample performance of three estimators for the correlation coefficient ρ_i defined in equation (6), vis-à-vis different dependence strength and data distribution.

5.1 Setting

We study the data-generating process in equations (2) and (3), where time-varying volatilities $\sigma_{jt}, j \in (i, m)$ are defined as in equation (8). Let w_{mt} and w_{it} be generated by *i.i.d.* normal and Student's t distribution with 4 degrees of freedom and consider a sample of size $T \in (1500, 3000)$. We consider the following set of parameters: $\tilde{\omega}_m = \tilde{\omega}_i = 0.1$, $\beta_m = \beta_i = 0.9$, $\tilde{\alpha}_m = \tilde{\alpha}_i = \tilde{\alpha}_{im} = 0.05$, $\rho_i \in (0.3, 0.6, 0.9)$.

Results are based on 1000 replications of the data-generating process. For each simulated sample, for $j \in (i, m)$ we first minimize the objective function as in (11) with $\theta = 0.01$ and then, using estimated parameter vector $\hat{\boldsymbol{\gamma}}_j$, we recover the standardized quantile specific

[8] See Appendix C for related analytical expressions.

residuals $\hat{\varepsilon}_{jt} = y_{jt}/\hat{q}_{jt}$ where q_{jt} is defined in equations (7). Then the least squares estimator of ρ_i is obtained as $\hat{\rho}_{OLS} = \arg\min_{\rho_i} T^{-1} \sum_{t=1}^{T} (\hat{\varepsilon}_{it} - \rho_i \hat{\varepsilon}_{mt})^2$. Along with the least squares estimator $\hat{\rho}_{OLS}$, we also consider the single-step Gaussian quasi-maximum likelihood estimator ρ_{QML} with time-varying conditional variances σ_{mt} and σ_{it} but constant conditional correlation ρ_i as in Bollerslev (1990).[9] We use the Nelder-Mead algorithm in the spirit of Engle and Manganelli (2004) in order to deal with estimation of the conditional quantile functions and the modified Newton-Raphson algorithm while dealing with the maximum likelihood estimation.[10]

[**Insert Tables 1 and 2 around here**]

5.2 Results

Tables 1 and 2 report the empirical mean, standard deviation (in brackets) and the root mean squared error (RMSE) (in square brackets) for each estimator defined earlier in this section. Results in Table 1 are based on the Gaussian data-generating process, whereas in Table 2 on the Student's t distribution. Due to the symmetry of innovation distribution we only focus on results for $\theta = 0.01$.

We note that the empirical mean of estimated correlation coefficients ρ_i is very close to true values across all estimators for each of Monte Carlo specifications and sample size. The empirical standard deviation and the RMSEs are slightly better for the QML estimate and tend to improve significantly as ρ increases (but the opposite is true for the RMSE of ρ^{OLS}). The improvements associated with the increased sample size can also be seen in values of the empirical standard deviation and the RMSEs. We also notice a slight deterioration in the results when one moves from Gaussian to Student's t innovations.

[9] For this model, referred to as the constant conditional correlation GARCH (CCC-GARCH) model, the Gaussian log-likelihood function is:

$$\mathcal{L}(\tilde{\gamma}) = c - (1/2) \sum_{t=1}^{T} \left(\ln|\sigma_{mt}^2| + \ln|\sigma_{it}^2| + \ln|\boldsymbol{P}| + \boldsymbol{y}_t' \boldsymbol{D}_t^{-1} \boldsymbol{P}^{-1} \boldsymbol{D}_t^{-1} \boldsymbol{y}_t \right),$$

where $\boldsymbol{D}_t = \text{diag}(\sigma_{mt}, \sigma_{it})$, $\boldsymbol{y}_t = (y_{mt}, y_{it})'$, $\tilde{\gamma} = (\tilde{\omega}_m, \beta_m, \tilde{\alpha}_m, \tilde{\omega}_i, \beta_i, \tilde{\alpha}_i, \tilde{\alpha}_{im}, \rho_i)'$ and $\boldsymbol{P}$ is 2×2 matrix with ones along the main diagonal and ρ_i as an off-diagonal element.

[10] All calculations are done either in Matlab and Stata.

6 Impact of the US equity market tail shocks

In this empirical application we illustrate our framework by measuring the impact of the equity market quantile shock and it's propagation dynamics to equity returns of selected US financial institutions.

6.1 Data description and computational issues

The empirical analysis is based on daily data taken from Datastream. We use the daily closing equity prices and transform it into the continuously compounded log returns covering the period from 3 January 2000 to 31 December 2014, for a sample of 3913 observations. We use the first 3413 observations for estimation and the last 500 observations for the out-of-sample testing purposes. The series selected for our empirical analysis include Standard & Poor 500 Index (S&P 500), J.P. Morgan (JPM), American Express (AXP) and U.S. Bancorp (USB), and report their descriptive statistics in Table 3. The sample skewness and kurtosis indicates deviations from the normal distribution. To deal with serial correlation in the S&P 500 index, we prefiltered the series by fitting an AR(6) model to the raw returns and used the corresponding residuals as our proxy for the market index y_{mt}.

[Insert Table 3 around here]

We use the Nelder-Mead simplex algorithm for regression quantile estimation in the spirit of Engle and Manganelli (2004).[11]

In constructing the covariance matrix for estimates of parameters $\boldsymbol{\gamma}$, following suggestions by Koenker (2005, p. 81) and Machado and Santos Silva (2013), as done by **WKM** we use the following bandwidth $\hat{h}_T = \hat{\kappa}_T[\phi^{-1}(\theta + c_T) - \phi^{-1}(\theta - c_T)]$ with

$$c_T = T^{-1/3}(\phi^{-1}(1-0.025)^{2/3}\left(\frac{1.5(\phi(\Phi^{-1}(\theta)))}{2(\Phi^{-1}(\theta))^2+1}\right)^{1/3},$$

[11]In particular, we generate $10^5 \times 7$ vector of independent Uniform$(-1, 1)$ random variables and select first $10^3 \times 7$ that yields the lowest criterion function as described in equation (11). Then each of those initial parameter values are used to initialize the simplex algorithm and to obtain the optimal parameter estimates. All calculations are done using either Stata or Matlab.

where $\phi(\cdot)$ and $\Phi(\cdot)$ are probability density and standard normal cumulative distribution functions, and $\hat{\kappa}_T$ is the median absolute deviation of the conditional quantile residual $u_{jt} = y_{jt} - q_{jt}, j \in (m, i)$.

6.2 Results

Table 4 reports parameter estimates for the conditional quantile functions $q_{jt}, j \in (m, i)$ at the $\theta = 1\%$ and $\theta = 99\%$ confidence levels for the market rate captured by the S&P 500 and the set of financial institutions represented by J.P. Morgan, U.S. Bancorp and American Express.

[**Insert Table 4 around here**]

We note that the conditional quantile functions evaluated at 1% and 99% confidence levels tend to be persistent processes with autoregressive coefficient values being in the vicinity of 0.9 uniformly across all series. The market news, captured by the absolute value of the market rate, is statistically significant for the left tail, implying both negative and positive news tend to amplify losses. This could be explained by well known empirical evidence that such news tend to amplify volatility as well. However, the market impact on the individual institutions becomes insignificant for the right tail, implying that the general market conditions play a minor role in determining the dynamic behaviour of the upper part of the distribution. Sensitivity of the conditional quantile functions to the own series is negative for the extreme left tails and positive for the right tail.

If model (1) represents the true DGP, the corresponding conditional quantile restriction must hold. We use the in-sample and out-of-sample dynamic quantile (DQ) tests by Engle and Manganelli (2004) for testing whether this is actually the case. For this purpose we use the one-step ahead forecast values of the conditional quantile functions $\hat{q}_{jt}$ and the fitted values of the centered quantile exceedances $\hat{\text{Hit}}_{jt} = I(y_{jt} - \hat{q}_{jt} < 0) - \theta, j \in (m, i)$. From Table 4 we see that the centered unconditional exceedences tend to be in the vicinity of zero both in-and out-of-sample and the out-of-sample DQ test performs well, exceptions being the left tail of UBS and the right tail of S&P 500.

[**Insert Figure 1 around here**]

Figure 1 plots the conditional quantile functions at $\theta = 1\%$ and $\theta = 99\%$ confidence levels. The severe spikes are observed around the period of the financial crisis and in particular around the default of Lehman Brothers. The quantiles of the individual financial institutions display higher amplification compared to the market index, reflecting the fact that risks are better diversified at the market level.

[**Insert Table 5 around here**]

As a specification test, we employ an autoregression of order five to study the extent of serial correlation in level and squared reduced form quantile shocks $\hat{\varepsilon}_{it}$ and $\hat{\varepsilon}_{mt}$ as well as in standardized residuals of the GARCH model in (8), and provide the test statistics for the joint significance of included lagged information. If our specification captures the relevant dynamics of the data, we should not find any autocorrelation in the reduced form quantile shocks. Table 5 reports p-values for these tests. The results successfully reject the significance of lagged information for shock levels, but we notice that serial correlation is stronger in squared shocks. The test rejects the null hypothesis that squared residuals are correlated in the case of the CCC-GARCH model and for some, but not all, the conditional quantile models. We deduce that the performance of the quantile specification is comparable, if not superior, to the one of the CCC-GARCH model.

[**Insert Table 6 around here**]

Table 6 reports the least squares and quasi-maximum likelihood estimates for the correlation parameter ρ_i in equation (6). To test the significance of the relationship (6), we test whether the least squares estimates $\hat{\rho}_i$ is significantly different from zero using t-test based on the results from Section 4. Estimates are highly statistically significant and indicate the presence of high degree co-movement between the reduced form individual and the structural market tail shocks. Interestingly, the estimates are close to CCC-GARCH estimates, though slightly higher for the confidence level $\theta = 1\%$ and lower for $\theta = 99\%$. In addition, since the correlation parameter is higher for the left tail shocks, the estimates provide strong evidence for the presence of asymmetric dependence.

[**Insert Figures 2, 3 and 4 around here**]

Figures 2 and 3 report the left and the right tail QIRFs for the set of financial institutions in response to the market quantile shock captured by the return level on the S&P 500 index being equal to its conditional quantile quantile value. We also provide the two standard deviation confidence bands. Several comments are in order. First, some set of financial institutions tend to react with a higher intensity compared to the S&P 500. This is in line with the comment to Figure 1, that the market index has lower volatility, as it embed the diversification effect associated with large portfolios. Second, as we have seen from the results in Table 4, the right tail of some institutions have insignificant loading on the lagged absolute value of the market rate. This fact is reflected in the QIRFs, because the impact is modest and statistically different from zero only for a few periods. Significance of the initial impact stems from the significant linear relationship between shocks reported in Table 6. Therefore, the estimates of the reduced-form conditional quantile functions does not necessarily capture the overall impact, as it would neglect the indirect impact that market shocks have on individual banks' returns. Third, the impact of a negative market shock is more pronounced and long-lasting compared to the impact of a positive market shock. This feature is well captured by the asymmetric QIRFs displayed in the figure 4. As we see, the impact of the left quantile shocks is more pronounced, and at times statistically significant despite the wide confidence intervals.

7 Conclusion

This article proposed an econometric framework for the definition and measurement of the impact of quantile shocks. We introduced a novel conditional quantile based data-generating process and derived the impulse response functions associated with quantile shocks, within the context of a Vector Autoregressive model for quantiles. An empirical application demonstrates the relevance of the new approach and uncovers asymmetric dynamics in the reaction of individual banks to market quantile shocks.

Appendices

A Proofs

A.1 Proof of Proposition 2

Combining the conditional quantile representation of the financial return process in equation (1) with the standard scale model in equations (2) - (3) we can write

$$\begin{aligned}
\varepsilon_{mt} &= \tfrac{y_{mt}}{q_{mt}} \\
&= \tfrac{\sigma_{mt} w_{mt}}{\sigma_{mt} F^{-1}(\theta)} \\
&= \tfrac{1}{F^{-1}(\theta)} w_{mt}, \\
\varepsilon_{it} &= \tfrac{1}{F^{-1}(\theta)} z_{it} \\
&= \tfrac{1}{F^{-1}(\theta)} \left(\rho_i w_{mt} + \sqrt{1-\rho_i^2} w_{it}\right) \\
&= \rho_i \varepsilon_{mt} + \tfrac{\sqrt{1-\rho_i^2}}{F^{-1}(\theta)} w_{it}, \quad \rho_i \in [-1, 1].
\end{aligned}$$

A.2 Proof of Theorem 1

For any $h \geq 1$, applying the Law of Iterated Expectations we can write

$$\begin{aligned}
\mathrm{E}_t\, \boldsymbol{q}_{t+h} &= \boldsymbol{\omega} + \mathrm{E}_t\left[\mathrm{E}_{t+h-1}\left[\boldsymbol{\Gamma}_{t+h-1}\right] \boldsymbol{q}_{t+h-1}\right] \\
&= \boldsymbol{\omega} + \boldsymbol{\Gamma}\, \mathrm{E}_t\, \boldsymbol{q}_{t+h-1} \\
&= \textstyle\sum_{l=0}^{h-1} \boldsymbol{\Gamma}^l \boldsymbol{\omega} + \boldsymbol{\Gamma}^{h-1}\, \mathrm{E}_t\, \boldsymbol{\Gamma}_t \boldsymbol{q}_t.
\end{aligned}$$

where $\boldsymbol{\Gamma} = \mathrm{E}_{t+h-1}\, \boldsymbol{\Gamma}_{t+h-1}$ and the results follows due to the *i.i.d.* property of $\boldsymbol{\Gamma}_t$.

Here we use the fact that due to the *i.i.d.* property of the data the conditional θ-quantile function q_{jt} has always the same sign. Then the QIRFs defined in Definition 4 have the following components:

$$\boldsymbol{\Gamma} = \begin{pmatrix} \beta_m + \alpha_m \operatorname{sgn}(q_{mt})\mu_m & 0 \\ \alpha_{im} \operatorname{sgn}(q_{mt})\mu_m & \beta_i + \alpha_i \operatorname{sgn}(q_{it})\mu_i \end{pmatrix},$$

$$\mathrm{E}_t\left[\boldsymbol{\Gamma}_t | \varepsilon_{mt} = 1\right] - \mathrm{E}_t\left[\boldsymbol{\Gamma}_t\right] = \begin{pmatrix} \alpha_m \operatorname{sgn}(q_{mt})(1-\mu_m) & 0 \\ \alpha_{im} \operatorname{sgn}(q_{mt})(1-\mu_m) & \alpha_i \operatorname{sgn}(q_{it})(\tilde{\mu}_i - \mu_i) \end{pmatrix},$$

where values μ_m, μ_i and $\tilde{\mu}_i$ are defined in Section (3).

B Results for the second step estimates

By standard results of two-step estimates (e.g., Newey and McFadden (1994)) the asymptotic variance of $\hat{\rho}_i$ can be obtained using mean value expansions around the true parameter ρ_i. Let $\hat{\psi}_{\rho,t} = \hat{\varepsilon}_{mt}(\hat{\varepsilon}_{it} - \hat{\rho}_i\hat{\varepsilon}_{mt})$, $\psi_{\rho,t} = \varepsilon_{mt}(\varepsilon_{it} - \rho_i\varepsilon_{mt})$ and $\bar{\psi}_{\rho,t} = \varepsilon_{mt}(\varepsilon_{it} - \bar{\rho}_i\varepsilon_{mt})$, where $\bar{\rho}_i$ lies between $\hat{\rho}_i$ and ρ_i, $\hat{\varepsilon}_{jt} = y_{jt}/\hat{q}_{jt}$ and $\varepsilon_{jt} = y_{jt}/q_{jt}$, $j \in (m,i)$. Then, given Corollary 1 we have:

$$\sqrt{T}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \;=\; \mathrm{diag}(\boldsymbol{H}_m^{-1}, \boldsymbol{H}_i^{-1})\; \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \begin{pmatrix} \nabla_{\boldsymbol{\gamma}_m} q_{mt}[\theta - I(y_{mt} < q_{mt})] \\ \nabla_{\boldsymbol{\gamma}_i} q_{it}[\theta - I(y_{it} < q_{it})] \end{pmatrix} + o_p(1),$$

and the straightforward manipulations yield

$$\begin{aligned}
\sqrt{T}(\hat{\rho}_i - \rho_i) &= (T^{-1}\textstyle\sum_{t=1}^T \hat{\varepsilon}_{mt}^2)^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^T (\hat{\varepsilon}_{it} - \rho_i \hat{\varepsilon}_{mt})\hat{\varepsilon}_{mt} \\
&= (T^{-1}\textstyle\sum_{t=1}^T \hat{\varepsilon}_{mt}^2)^{-1} \\
&\times \tfrac{1}{\sqrt{T}} \textstyle\sum_{t=1}^T \left[\psi_{\rho t} + \nabla_{\boldsymbol{\gamma}'}\bar{\psi}_{\rho t}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma})\right] \\
&= \left[(T^{-1}\textstyle\sum_{t=1}^T \hat{\varepsilon}_{mt}^2)^{-1}, \quad (T^{-1}\sum_{t=1}^T \hat{\varepsilon}_{mt}^2)^{-1}(T^{-1}\sum_{t=1}^T \nabla_{\boldsymbol{\gamma}'}\bar{\psi}_{\rho t})\right] \\
&\times \begin{pmatrix} \frac{1}{\sqrt{T}} \sum_{t=1}^T \psi_{\rho t} \\ \sqrt{T}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \end{pmatrix} \\
&= \left(H_\rho^{-1}, \quad H_\rho^{-1}\boldsymbol{H}_{\rho,\gamma}\boldsymbol{H}^{-1}\right) \tfrac{1}{\sqrt{T}} \textstyle\sum_{t=1}^T \begin{pmatrix} \psi_{\rho t} \\ \nabla_{\boldsymbol{\gamma}_m} q_{mt}[\theta - I(y_{mt} < q_{mt})] \\ \nabla_{\boldsymbol{\gamma}_i} q_{it}[\theta - I(y_{it} < q_{it})] \end{pmatrix} + o_p(1),
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{H}^{-1} &= \mathrm{diag}(\boldsymbol{H}_m^{-1}, \boldsymbol{H}_i^{-1}), \\
\boldsymbol{H}_{\rho,\boldsymbol{\gamma}} &= \mathrm{E}\,\nabla_{\boldsymbol{\gamma}'}\bar{\psi}_{\rho t} \\
&= \mathrm{E}\left[\; (2\rho_i\varepsilon_{mt} - \varepsilon_{it})\, \tfrac{\varepsilon_{mt}}{q_{mt}} \nabla_{\boldsymbol{\gamma}_m'} q_{mt}, -\tfrac{\varepsilon_{it}\varepsilon_{mt}}{q_{it}} \nabla_{\boldsymbol{\gamma}_i'} q_{it} \;\right], \\
H_\rho &= \mathrm{E}\,\varepsilon_{mt}^2.
\end{aligned}$$

Then $\hat{\rho}_i$ is consistent, due to consistency of $\hat{\boldsymbol{\gamma}}$ shown in Corollary (1) and independence of ε_{mt} and ω_{it} imposed in Definition 2, and asymptotically normal

$$\sqrt{T}(\hat{\rho}_i - \rho_i) \sim \mathrm{N}(0, \boldsymbol{H}_\rho \boldsymbol{J}_\rho \boldsymbol{H}_\rho'),$$

where

$$\begin{aligned}
\boldsymbol{J}_\rho &= \mathrm{E}\left(\boldsymbol{\psi}_t \boldsymbol{\psi}_t'\right), \\
\boldsymbol{\psi}_t &= \left(\begin{array}{c} (\varepsilon_{it} - \rho_i \varepsilon_{mt})\varepsilon_{mt} \\ \nabla_{\boldsymbol{\gamma}_m} q_{mt}[\theta - I(y_{mt} < q_{mt})] \\ \nabla_{\boldsymbol{\gamma}_i} q_{it}[\theta - I(y_{it} < q_{it})] \end{array} \right), \\
\boldsymbol{H}_\rho &= \left(H_\rho^{-1}, H_\rho^{-1} \boldsymbol{H}_{\rho,\gamma} \boldsymbol{H}^{-1}\right).
\end{aligned}$$

The proof of asymptotic distribution for estimates $\hat{\mu}_j, j \in (m,i)$ and $\hat{\tilde{\mu}}_i$ follows easily using similar arguments to those discussed above and therefore is omitted.

C Analytic derivations

Here we provide analytic expressions for the components of $\nabla_{\boldsymbol{\gamma}'} \boldsymbol{\Delta}^h$. Note that for any parameter $\tilde{\gamma}$ from the parameter vector $\boldsymbol{\gamma}$ we have

$$\tfrac{\partial}{\partial \tilde{\gamma}} \boldsymbol{\Gamma}^l = \left(\tfrac{\partial}{\partial \tilde{\gamma}} \boldsymbol{\Gamma}\right) \boldsymbol{\Gamma}^{l-1} + \boldsymbol{\Gamma} \tfrac{\partial}{\partial \tilde{\gamma}} \boldsymbol{\Gamma}^{l-1}, \quad l = 2, \ldots, h.$$

Let $\boldsymbol{\Delta}_0 = \mathrm{E}_t\left[\boldsymbol{\Gamma}_t | \varepsilon_{mt} = 1\right] - \mathrm{E}_t[\boldsymbol{\Gamma}_t]$ and $\boldsymbol{\nu} = (\boldsymbol{\gamma}_m', \boldsymbol{\gamma}_i', \mu_m, \mu_i, \tilde{\mu}_i)'$. Then, for the left tail QIRFs we have

$$\begin{aligned}
\nabla_{\boldsymbol{\nu}'} \boldsymbol{\Gamma} &= \left[\tfrac{\partial}{\partial \omega_j} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \beta_m} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \alpha_m} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \omega_i} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \beta_i} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \alpha_i} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \alpha_{im}} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \mu_m} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \mu_i} \boldsymbol{\Gamma}, \tfrac{\partial}{\partial \tilde{\mu}_i} \boldsymbol{\Gamma}\right] \\
&= \left[\mathbf{0}, \begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix}, \begin{matrix} -\mu_m & 0 \\ 0 & 0 \end{matrix}, \mathbf{0}, \begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & -\mu_i \end{matrix}, \begin{matrix} 0 & 0 \\ -\mu_m & 0 \end{matrix}, \begin{matrix} -\alpha_m & 0 \\ -\alpha_{im} & 0 \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & -\alpha_i \end{matrix}, \mathbf{0}\right], \\
\nabla_{\boldsymbol{\nu}'} \boldsymbol{\Delta}_0 &= \left[\mathbf{0}, \mathbf{0}, \begin{matrix} \mu_m - 1 & 0 \\ 0 & 0 \end{matrix}, \mathbf{0}, \mathbf{0}, \begin{matrix} 0 & 0 \\ 0 & \mu_i - \tilde{\mu}_i \end{matrix}, \begin{matrix} 0 & 0 \\ \mu_m - 1 & 0 \end{matrix}, \begin{matrix} \alpha_m & 0 \\ \alpha_{im} & 0 \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & \alpha_i \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & -\alpha_i \end{matrix}\right].
\end{aligned}$$

Similarly, we can provide the analytic results for the right tail as following

$$\begin{aligned}
\nabla_{\boldsymbol{\nu}'} \boldsymbol{\Gamma} &= \left[\mathbf{0}, \begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix}, \begin{matrix} \mu_m & 0 \\ 0 & 0 \end{matrix}, \mathbf{0}, \begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & \mu_i \end{matrix}, \begin{matrix} 0 & 0 \\ \mu_m & 0 \end{matrix}, \begin{matrix} \alpha_m & 0 \\ \alpha_{im} & 0 \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & \alpha_i \end{matrix}, \mathbf{0}\right] \\
\nabla_{\boldsymbol{\nu}'} \boldsymbol{\Delta}_0 &= \left[\mathbf{0}, \mathbf{0}, \begin{matrix} 1 - \mu_m & 0 \\ 0 & 0 \end{matrix}, \mathbf{0}, \mathbf{0}, \begin{matrix} 0 & 0 \\ 0 & \tilde{\mu}_i - \mu_i \end{matrix}, \begin{matrix} 0 & 0 \\ 1 - \mu_m & 0 \end{matrix}, \begin{matrix} -\alpha_m & 0 \\ -\alpha_{im} & 0 \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & -\alpha_i \end{matrix}, \begin{matrix} 0 & 0 \\ 0 & \alpha_i \end{matrix}\right].
\end{aligned}$$

Furthermore, for $j \in (m,i)$ consider the following analytic derivatives

$$\frac{\partial}{\partial \omega_j} q_{jt} = \begin{cases} 1 + \beta_j \frac{\partial}{\partial \omega_j} q_{jt-1}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \beta_j} q_{jt} = \begin{cases} q_{jt-1} + \beta_j \frac{\partial}{\partial \beta_j} q_{jt-1}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

$$\frac{\partial}{\partial \alpha_j} q_{jt} = \begin{cases} |y_{jt-1}| + \beta_j \frac{\partial}{\partial \alpha_j} q_{jt-1}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

and

$$\frac{\partial}{\partial \alpha_{im}} q_{it} = \begin{cases} |y_{mt-1}| + \beta_i \frac{\partial}{\partial \alpha_{im}} q_{it-1}, & \text{for} \quad t \geq 2, \\ 0, & \text{for} \quad t = 1, \end{cases}$$

where we make use of fact that dynamic quantiles are initialized using the empirical quantile of first T_0 observations.[12]

[12] In empirical application we set $T_0 = 100$.

D Empirical results

Table 1: Simulation with Gaussian innovations.

	$\theta = 0.01, \tilde{\omega}_m = \tilde{\omega}_i = 0.1,\ \beta_m = \beta_i = 0.9,\ \tilde{\alpha}_m = \tilde{\alpha}_i = \tilde{\alpha}_{im} = 0.05.$			
	$w_{mt}, w_{it} \sim i.i.d.\mathrm{N}(0,1).$			
	T=1500		T=3000	
ρ	ρ_{OLS}	ρ_{QML}	ρ_{OLS}	ρ_{QML}
	0.2988	0.3029	0.2983	0.3008
0.3	(0.0282)	(0.0240)	(0.0197)	(0.0172)
	[0.0282]	[0.0241]	[0.0197]	[0.0172]
	0.5978	0.6030	0.5977	0.6012
0.6	(0.0339)	(0.0166)	(0.0232)	(0.0121)
	[0.0340]	[0.0168]	[0.0233]	[0.0122]
	0.8953	0.9015	0.8978	0.9008
0.9	(0.0352)	(0.0048)	(0.0241)	(0.0036)
	[0.0355]	[0.0051]	[0.0241]	[0.0036]

Note: The table reports the finite sample evidence for the correlation coefficient ρ_i using the least squares (ρ_{OLS}) and the quasi-maximum likelihood (ρ_{QML}) estimates, where ρ_{QML} is based on the Gaussian log-likelihood function. The empirical mean, standard deviation (in brackets) and root mean squared error (in square brackets) for each estimator are reported.

Table 2: Simulation with Student-t innovations

	$\theta = 0.01, \tilde{\omega}_m = \tilde{\omega}_i = 0.1,\ \beta_m = \beta_i = 0.9,\ \tilde{\alpha}_m = \tilde{\alpha}_i = \tilde{\alpha}_{im} = 0.05.$			
	$w_{mt}, w_{it} \sim i.i.d.\mathrm{t}(4).$			
	T=1500		T=3000	
ρ	ρ_{OLS}	ρ_{QML}	ρ_{OLS}	ρ_{QML}
	0.2928	0.3018	0.2946	0.3014
0.3	(0.0469)	(0.0285)	(0.0369)	(0.0200)
	[0.0474]	[0.0286]	[0.0372]	[0.0200]
	0.5740	0.6025	0.5783	0.6018
0.6	(0.0609)	(0.0281)	(0.0439)	(0.0194)
	[0.0661]	[0.0282]	[0.0489]	[0.0195]
	0.8662	0.9010	0.8742	0.9007
0.9	(0.0539)	(0.0113)	(0.0387)	(0.0078)
	[0.0636]	[0.0114]	[0.0465]	[0.0078]

Note: The table reports the finite sample evidence for the correlation coefficient ρ_i using the least squares (ρ_{OLS}) and the quasi-maximum likelihood (ρ_{QML}) estimates, where ρ_{QML} is based on the Gaussian log-likelihood function. The empirical mean, standard deviation (in brackets) and root mean squared error (in square brackets) for each estimator are reported.

Table 3: Descriptive statistics for equity returns

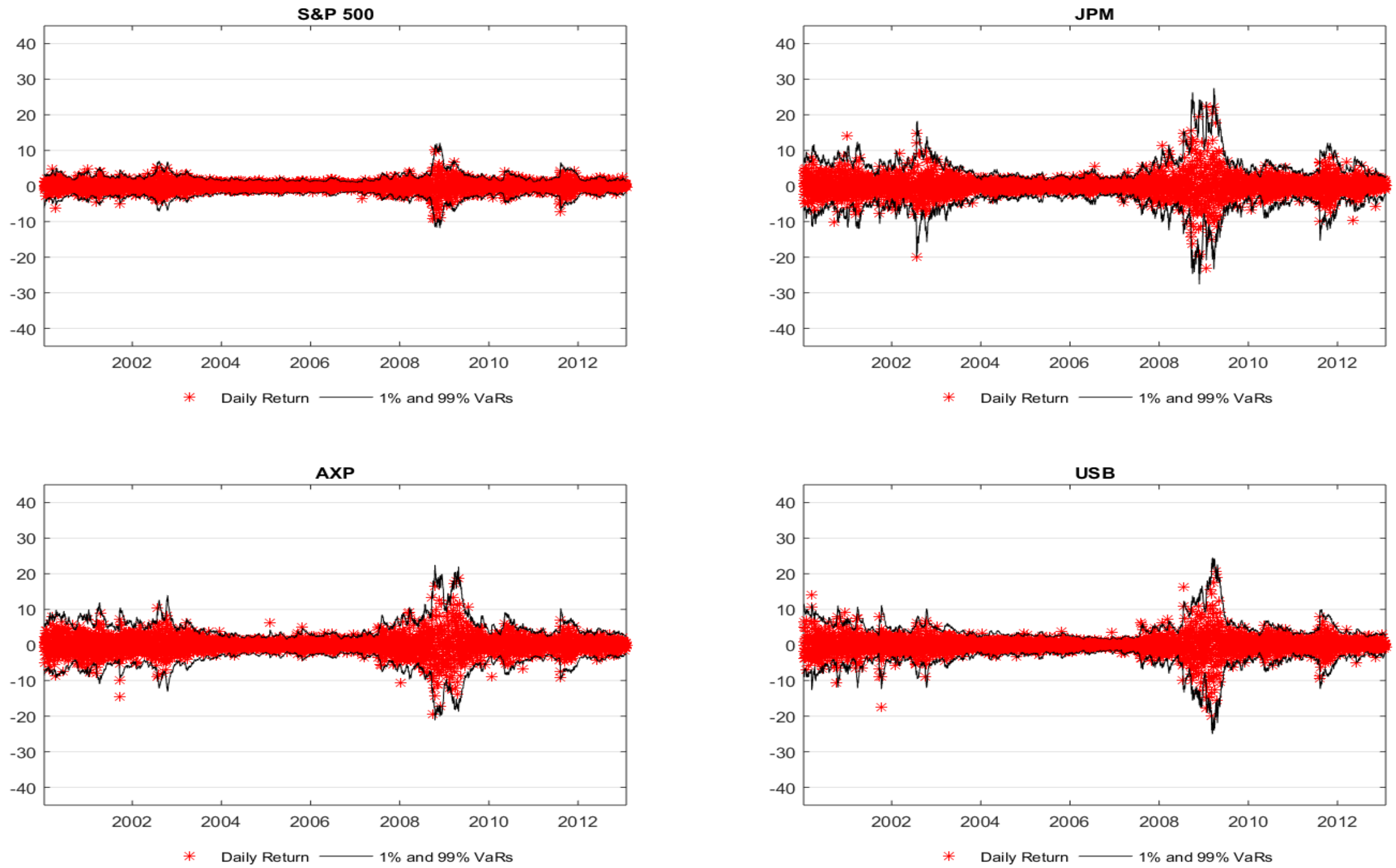
	Mean	Std. Dev.	Max	Min	Skewness	Kurtosis
S&P 500	0.0000	1.3134	10.2282	-9.1455	-0.3411	10.2934
JPM	-0.0003	2.7799	22.3848	-23.2257	0.2653	14.4935
AXP	0.0081	2.5083	18.7607	-19.3595	0.0002	11.4304
USB	0.0130	2.4336	20.5734	-20.0514	-0.0541	15.5924

Note: The table reports the descriptive statistics of the daily equity returns for the United States based financial institutions and the S&P 500 index. Statistics for S&P 500 series is presented after pre-filtering using autoregression of order six. The sample ranges from January 10, 2000, to January 29, 2012.

Table 4: Parameter estimates for dynamic conditional quantile functions

	$\theta = 0.01$						$\theta = 0.99$				
	S&P500		JPM	AXP	USB		S&P500		JPM	AXP	USB
	-0.0756		-0.0917	-0.0270	-0.0422		0.02084		0.0473	0.0422	0.0466
ω_m	(0.0281)	ω_i	(0.0439)	(0.0172)	(0.0188)	ω_m	(0.01822)	ω_i	(0.0192)	(0.0355)	(0.0527)
	[0.0071]		[0.0365]	[0.1159]	[0.0247]		[0.25258]		[0.0139]	[0.2339]	[0.3762]
	0.9050		0.8512	0.9286	0.8876		0.91466		0.9070	0.9106	0.9249
β_m	(0.0195)	β_i	(0.0296)	(0.0144)	(0.0159)	β_m	(0.02680)	β_i	(0.0077)	(0.0253)	(0.0339)
	[0.0000]		[0.0000]	[0.0000]	[0.0000]		[0.0000]		[0.0000]	[0.0000]	[0.0000]
	-0.2444		-0.2453	-0.1575	-0.2735		0.24079		0.3136	0.2579	0.2102
α_m	(0.0402)	α_i	(0.0502)	(0.0457)	(0.0327)	α_m	(0.08192)	α_i	(0.0374)	(0.0687)	(0.0604)
	[0.0000]		[0.0000]	[0.0006]	[0.0000]		[0.00329]		[0.0000]	[0.0002]	[0.0005]
			-0.4012	-0.1192	-0.1151				-0.0391	0.0381	0.0179
		α_{im}	(0.2198)	(0.0524)	(0.0608)			α_{im}	(0.0416)	(0.0730)	(0.0710)
			[0.0680]	[0.0230]	[0.0584]				[0.34710]	[0.6016]	[0.8005]
$\mathcal{RQ}$	128.9529		267.3816	239.3424	229.8153		105.6480		255.1507	216.2181	215.6272
Hits in-sample	0.0000		-0.0003	0.0000	0.0000		0.0000		0.0003	-0.0003	0.0000
Hits out-of-sample	-0.0020		-0.0060	-0.0060	0.0040		0.0000		0.0040	0.0000	0.0040
$\mathcal{DQ}$ in-sample	0.7138		0.4741	0.4160	0.2581		0.8246		0.4742	0.4810	0.5230
$\mathcal{DQ}$ out-of-sample	0.5137		0.3856	0.5955	0.0000		0.0000		0.5072	0.9400	0.7238

Note: Estimates correspond to the system of conditional quantile functions in equation (7) at $\theta = 1\%$ and $\theta = 99\%$ confidence levels. Standard deviations are reported in parentheses and p-values are in square brackets. $\mathcal{RQ}$ corresponds to the value of the objective function in equation (11) and $\mathcal{DQ}$ stands for p-values of the dynamic quantile test by Engle and Manganelli (2004). The hit values stand for unconditional means of centered exceedances.



Note: The figure plots estimates of the conditional quantile functions $q_{jt}, j \in (m, i)$ at $\theta = 1\%$ and $\theta = 99\%$ confidence levels using the specification in equation (7). The amplification corresponds to the financial crisis triggered by the Lehman Brothers default.

Figure 1: Estimates of the conditional quantile functions

Table 5: Persistence of the standardized quantile and CCC-GARCH shocks.

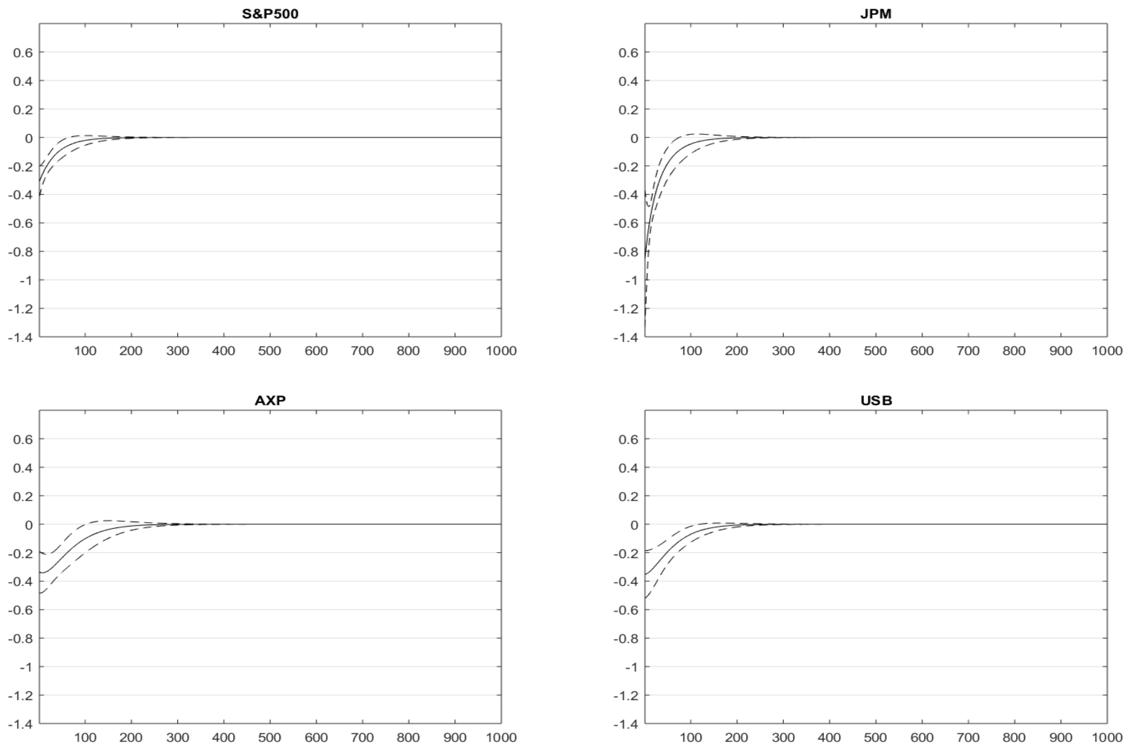
	$\hat{\varepsilon}_t$			$\hat{\varepsilon}_t^2$		
	$\theta = 0.01$	$\theta = 0.99$	QML	$\theta = 0.01$	$\theta = 0.99$	QML
S&P500	0.2190	0.2266	0.2275	0.0000	0.0076	0.0000
JPM	0.5137	0.3955	0.3671	0.8852	0.2577	0.0005
AXP	0.0799	0.1608	0.0625	0.0106	0.2447	0.0000
USB	0.8038	0.6588	0.7914	0.0650	0.0000	0.0002

Note: The table reports p-values for statistical significance of included lagged information in AR(5) model for series of standardized conditional quantile and QML shocks $\hat{\varepsilon}_t$ and their squared levels $\hat{\varepsilon}_t^2$.

Table 6: Estimates of the correlation coefficient ρ_i

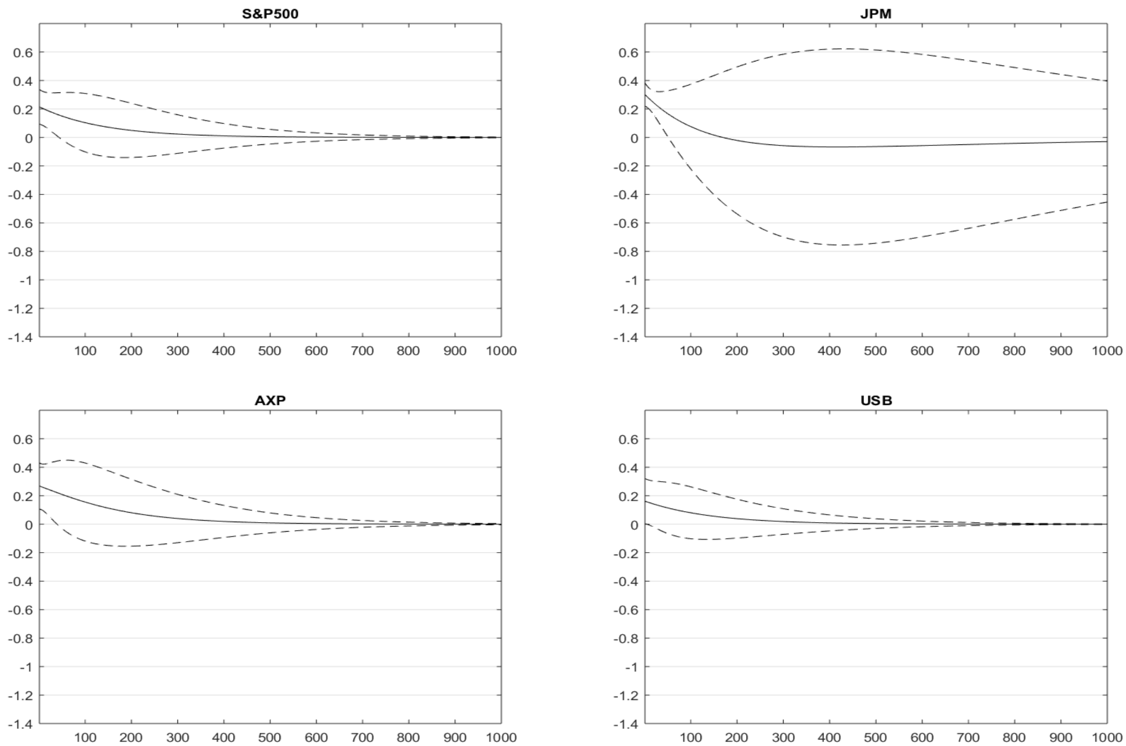
	$\theta = 0.01$				$\theta = 0.99$		
	JPM	AXP	USB		JPM	AXP	USB
	0.7858	0.7709	0.7080		.6668	.6568	0.5870
ρ_{OLS}	(0.0268)	(0.0239)	(0.0271)		(0.0238)	(0.0228)	(0.0278)
	[0.0000]	[0.0000]	[0.0000]		[0.0000]	[0.0000]	[0.0000]
ς_i	0.2674	0.2706	0.3024		0.2683	0.2731	0.2941
R^2	0.5460	0.5306	0.4330		0.5378	0.5214	0.4287
RSS	243.4728	249.3861	311.3110		245.2572	254.0463	294.7258
			0.7393	0.7326	0.6587		
ρ_{QML}			(0.0103)	(0.0100)	(0.0129)		
			[0.0000]	[0.0000]	[0.0000]		
Log-likelihood			-4592.2564	-4424.6085	-4347.0107		

Note: The parameter estimates correspond to the correlation among standardized quantile residuals of the market ε_{mt} and individual financial institution ε_{it} using the specification in equation (6). Estimates are based on the ordinary least squares ρ_{OLS} and the constant conditional correlation CCC-GARCH model of Bollerslev (1990) ρ_{QML} based on the Gaussian likelihood function. Standard errors are reported in brackets and p-values in square brackets. RSS and R^2 stand for the residual sum of squares and the R-squared statistics from the least squares regression.



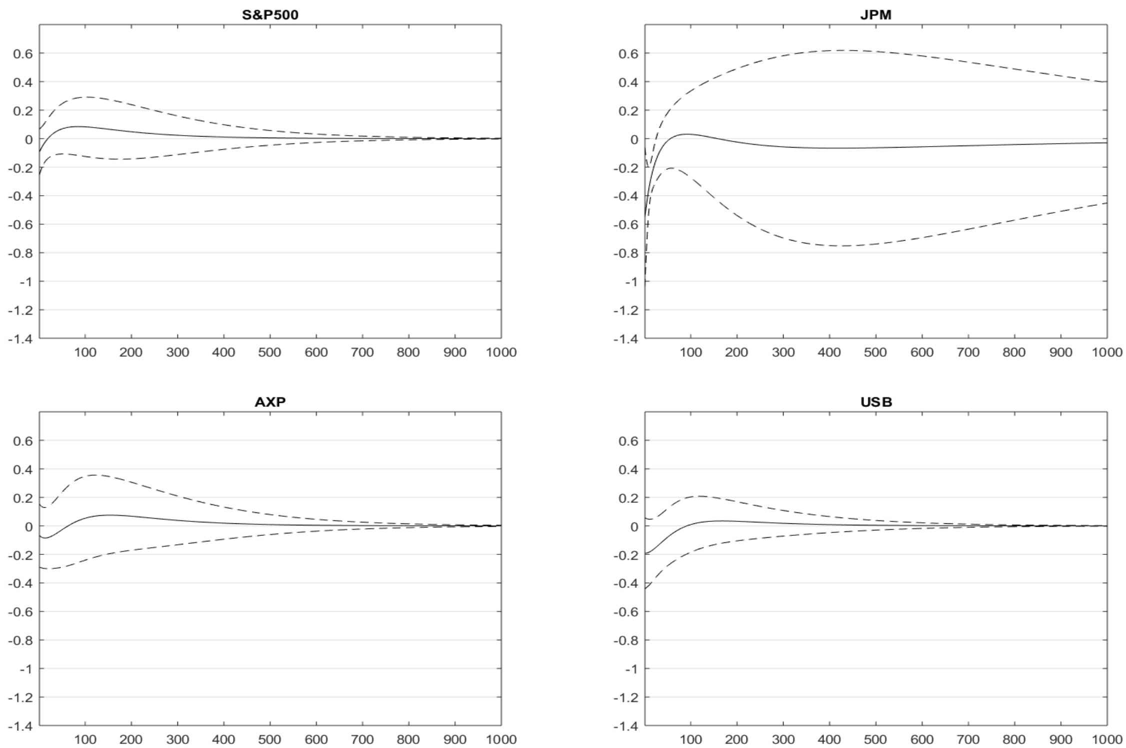
Note: The charts plot the quantile impulse-response functions of a market tail shock at $\theta = 1\%$ confidence level, together with two standard deviation confidence bands. The responses are calculated using the specification where the shock to a financial institution is contemporaneously affected by the structural market quantile shock.

Figure 2: Quantile impulse-response functions to a shock to the market rate



Note: The charts plot the quantile impulse-response functions of a market tail shock at $\theta = 99\%$ confidence level, together with two standard deviation confidence bands. The responses are calculated using the specification where the shock to a financial institution is contemporaneously affected by the structural market quantile shock.

Figure 3: Quantile impulse-response functions to a shock to the market rate



Note: The charts plot the asymmetric quantile impulse-response functions of a market tail shock at $\theta = 99\%$ and $\theta = 1\%$ confidence levels, together with two standard deviation confidence bands. The responses are calculated using the specification where the shock to a financial institution is contemporaneously affected by the structural market quantile shock.

Figure 4: Asymmetric quantile impulse-response functions to a shock to the market rate

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